

DERIVATION OF SEASONDE RADIAL VELOCITIES

Lipa and Barrick [1] described the basic methods for the analysis of broad-beam HF radar Doppler echo spectra to give radial current velocities. We here summarize the analysis procedures presently implemented to derive radial velocities from SeaSonde radar spectra.

Summary of analysis procedures

SeaSonde system software performs the following eight steps on the complex voltage time series obtained from by the three antennas:

- (i) The complex signal voltages from the three SeaSonde antennas, V_i , $i=1,2,3$, for each range cell are combined to give the voltage cross spectra defined by $V_i V_j^*$ where $i, j = 1,2,3$. A range cell is a circular annulus defined by the range from the radar and a fixed range increment that is typically 1.5, 3, 10 km for transmit frequencies in the 24-27, 12-14, 4-6 MHz bands.
- (ii) The voltage cross spectra are averaged over a time interval, which is usually set to 10 minutes for standard-range SeaSonde (with transmit frequency in either the 12-14 MHz or the 24-27 MHz band), 30 minutes for a long-range SeaSonde (transmit frequency in the 4-6 MHz band). The averaged spectra are denoted as $\langle V_i V_j^* \rangle$.
- (iii) The radial current velocity corresponding to a given signal frequency is calculated. The velocity is proportional to the frequency difference from the ideal Bragg frequency.
- (iv) Boundaries on the radar spectrum delimiting the region due to first-order scatter from the sea are defined. Empirical methods are used to separate the first-order spectrum surrounding the ideal Bragg frequencies from the neighboring lower-amplitude, second-order structure and noise. Note that if the first-order region is set too wide, the anomalous region included at the outer edges will lead to large, incorrect current velocity vectors.
- (v) Empirical methods are used to judge if a first-order spectral region is contaminated by radar interference, in which case it is excluded from further analysis.
- (vi) The voltage cross spectra are then analyzed using the MUSIC algorithm [2,3] to obtain the direction of arrival of the signal, using either ideal or measured antenna patterns. If ideal patterns are used, they are first corrected for phase and amplitude mismatches between the loop antennas and the monopole. Further detail on MUSIC is given below. This calculation results in the directions of arrival of the signal for each value of the radial velocity and for all range cells.
- (vii) The analysis described in the Step (vi) typically produces several velocity values in a given radar cell, which is a segment of a range cell defined by the azimuth angle from the radar and azimuth increment, illustrated schematically

- in Fig.1. These values are averaged to give the final output value for that location and the standard deviation is calculated.
- (viii) The results are then time averaged (merged) over seven consecutive radial maps.

SeaSonde direction-finding with MUSIC

The MUSIC algorithm was first presented by Schmidt [2] in 1986. Barrick and Lipa [3] described its application to SeaSonde data. We here give a summary of the algorithm as it is implemented in SeaSonde software.

- (i) Voltages V_1, V_2, V_3 from the antennas are modeled as the product of the complex antenna pattern values and the complex echo signal amplitudes. The signal at a given Doppler shift (and hence a given radial velocity) is assumed to come from at most two directions. This limit is imposed by the information available from the three antennas.

Assuming that the radar echo from the sea comes from a single direction φ_1 , we write the complex zero-mean voltages as a 3x1 matrix formed as the product of the antenna pattern matrix $[a]$ at that bearing and the signal amplitude s_1 :

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = [a] s_1 \quad \text{where} \quad [a] = \begin{bmatrix} a_1(\varphi_1) \\ a_2(\varphi_1) \\ a_3(\varphi_1) \end{bmatrix} \quad (1)$$

Assuming that the sea-echo comes from two directions φ_1, φ_2 , the corresponding equation is given by:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = [a] \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \quad \text{where} \quad [a] = \begin{bmatrix} a_1(\varphi_1) & a_1(\varphi_2) \\ a_2(\varphi_1) & a_2(\varphi_2) \\ a_3(\varphi_1) & a_3(\varphi_2) \end{bmatrix} \quad (2)$$

where s_1 and s_2 are the signal amplitudes from the two directions.

- (ii) At a given Doppler frequency, we form the covariance matrix C of the complex signal voltages from the three antennas, a 3x3 complex Hermitian matrix:

$$C = \begin{bmatrix} V_1 V_1^* & V_1 V_2^* & V_1 V_3^* \\ V_2 V_1^* & V_2 V_2^* & V_2 V_3^* \\ V_3 V_1^* & V_3 V_2^* & V_3 V_3^* \end{bmatrix} \quad (3)$$

Substituting (2) into (3) and taking the time average gives:

$$\langle C \rangle = \begin{bmatrix} \langle V_1 V_1^* \rangle & \langle V_1 V_2^* \rangle & \langle V_1 V_3^* \rangle \\ \langle V_2 V_1^* \rangle & \langle V_2 V_2^* \rangle & \langle V_2 V_3^* \rangle \\ \langle V_3 V_1^* \rangle & \langle V_3 V_2^* \rangle & \langle V_3 V_3^* \rangle \end{bmatrix} = [a][S][a^*]^T \quad (4)$$

where T denotes the transpose, the elements of $\langle C \rangle$ represent the SeaSonde voltage cross spectra, and S is given for a signal from a single direction by:

$$S = \langle s_1 s_1^* \rangle \quad (5)$$

and for a signal from two directions by:

$$S = \begin{bmatrix} \langle s_1 s_1^* \rangle & \langle s_1 s_2^* \rangle \\ \langle s_2 s_1^* \rangle & \langle s_2 s_2^* \rangle \end{bmatrix} \quad (6)$$

Ideally, $s_1 s_2^*$ and $s_2 s_1^*$ in (6) average to zero because sea-echo signals are uncorrelated for angular separations as small as 0.5° , as shown by Barrick and Snyder [4]. This is less than the angular spacing of the radar cells,

- (iii) An eigenfunction analysis is performed on the covariance matrix. The largest eigenvalues and their corresponding eigenvectors represent the sea echo, whereas the smaller eigenvalues represent noise. When the signal is from two/one directions, ideally there are two/one nonzero eigenvalues. In practice, the noise eigenvalues are finite but small compared with the signal eigenvalues.
- (iv) The direction(s) of arrival of the signal(s) are determined using the fundamental principle behind MUSIC: the signal eigenvector at the correct bearing is orthogonal to all the noise eigenvectors. The algorithm finds the angle(s) at which this occurs. This procedure is carried out first assuming that the signals are coming from two directions, to give the two angles and the 2x2 signal matrix, termed ‘dual-angle solutions’. Then it is assumed that the signal comes from only one direction. The calculation gives the single optimum angle and the corresponding signal power, termed a ‘single-angle’ solution.
- (v) The dual-angle solution is then tested to judge its validity. Three criteria are defined based on the following observations: (1) If the echo signal is indeed coming from two directions, the two signal eigenvalues and the corresponding values of signal power will be much greater than the noise values. (2) The two signal eigenvalues and powers should be reasonably close in magnitude, as if one signal power or eigenvalue is far less than the other, it follows that most energy comes from a single direction which would cast doubt on the dual-angle solution. (3) As discussed previously, the off-diagonal

elements of the signal matrix S are ideally zero and in practice should be much smaller than the diagonal elements for a valid dual-angle solution. Defining three parameters P_1 , P_2 , P_3 (termed the MUSIC parameter set), these observations lead to the following three criteria for a valid dual-angle solution:

1. The ratio of the largest covariance matrix eigenvalue to the second-largest must be less than P_1 .
2. The ratio of the largest of two signal powers $\langle s_1 s_1^* \rangle$ and $\langle s_2 s_2^* \rangle$ to the smallest must be less than P_2 .
3. For the signal matrix S defined by (6), the ratio of the product of the diagonal elements to the product of the off-diagonal elements must be greater than P_3

If any of these tests are failed by the dual-angle solution, it is rejected and the single angle solution is accepted.

To illustrate when single/dual angle solutions apply, we note that for the case of a uniform current flowing parallel to a straight shore, single-angle solutions will be produced at every azimuth around a range cell, as the radial component of the velocity is then a single-valued function of azimuth. In the unlikely situation that a uniform current flows perpendicular to a straight coast, all the solutions would be dual angle except for the azimuth normal to the coastline, as the radial velocity components occur in pairs around the range cell. Ideally, the parameter set would produce a percentage of dual angle solutions which is consistent with that calculated from the derived radial current map. With the software presently in use, it is necessary to preset the MUSIC parameters to fixed values, although obviously a single parameter set may not be appropriate for all current velocity patterns as the percentage of dual-angle solutions varies with time and location.

To demonstrate the proportion of dual-angle solutions in a typical measurement, the percentage of dual-angle solutions obtained for three different MUSIC parameter sets are plotted in Fig. 1 for a data set consisting of seven days of 10-minute radar spectra measured by the Seasonde located at Misquamicut, RI. The entire first-order region (for both positive and negative Doppler frequencies) and for all range cells were analyzed to give radial velocities. Results for the dual-angle percentages are plotted in blue, red, green for the MUSIC parameter sets [30, 15, 1.8], [20,10,3], [10, 5, 8]. For the standard SeaSonde parameter set [20,10,3] in use until recently, there are typically less than 20% dual-angle solutions. This parameter set was chosen by simulating typical current scenarios and minimizing the differences between the input currents and the calculated values. It has recently been replaced with the set [40,20,2], which yields more dual-angle solutions and fewer gaps in the radial map.

REFERENCES

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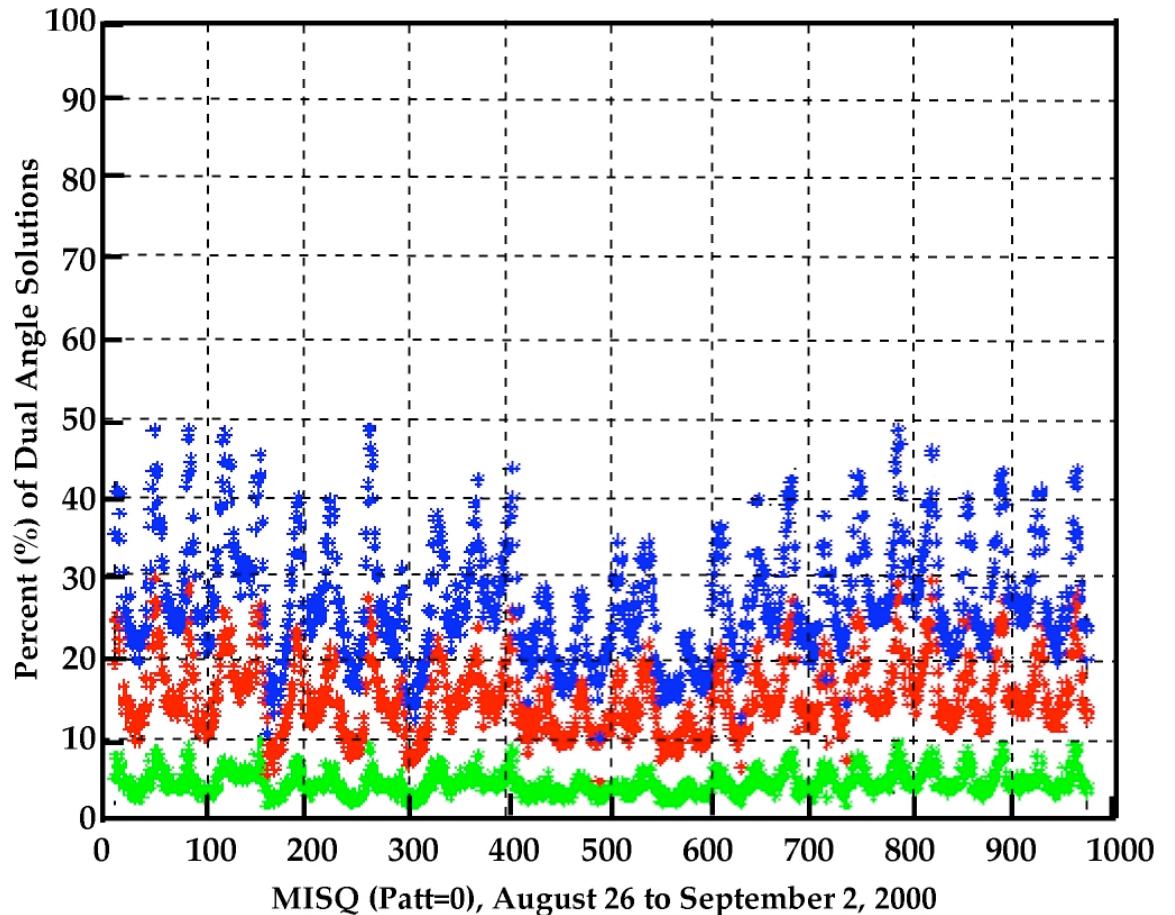


Fig. 1. Percentage of Doppler points that yield a dual-angle solution vs. run number for different MUSIC parameter sets $[P_1, P_2, P_3]$, Blue [30, 15, 1.8], Red [20, 10, 3], Green [10, 5, 8] plotted versus the run number. Each run consists of analysis of a 10-minute averaged spectra from Misquamicut measured from August 26-September 2, 2000. Ideal antenna patterns used in analysis.