The 10-Minute Introduction to

Direction-Finding
vs
Beam Forming
Applied to
HF Radar
Coastal Current Mapping

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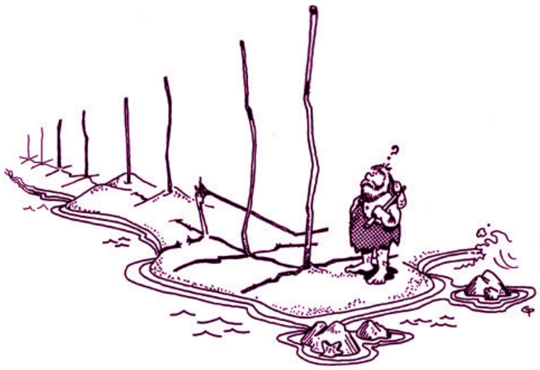
SUMMARY: The Best Technology Evolves and Proliferates



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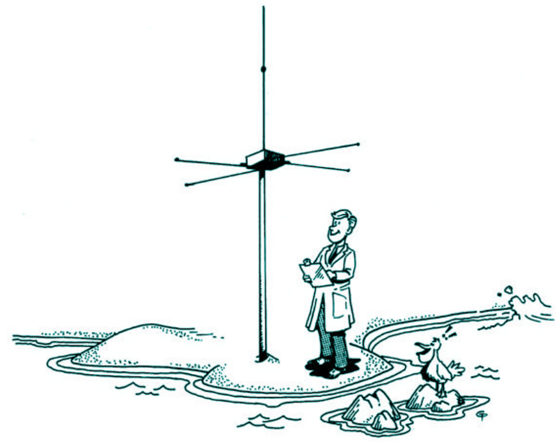

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- **The SeaSonde crossed-loop/monopole receive antennas indeed measure angles accurately in the mapping of sea surface currents.**
- For the sine, cosine, and constant antenna patterns (vs bearing angle), we illustrate this using the simple Arctangent function performed in software.
- For actual measured patterns that deviate from these ideals the more robust MUSIC algorithm is used and handles this direction-finding task very robustly.
- This process is applied independently at each of the many echo-signal Doppler frequencies induced by the radial current pattern observed within each radar range ring.
- The algorithm uniquely resolves up to two different directions over 360° that go with each Doppler spectral frequency.
- Since there are two Doppler bins for each of two independent Bragg echo peaks, there can be up to four directions resolved at each radial speed; this normally covers even the most complex flow fields one could expect to encounter.
- There is no hydrodynamic model fitted to the data, nor any hidden assumptions.
- The inherent speed resolution dictated by the SeaSonde Doppler-bin width is 4.7 cm/s at 12 MHz; at the 25 MHz operating frequency, this speed resolution becomes 2.3 cm/s.
- The typical statistical (rms) bearing error due to noise in the SeaSonde (based on one-hour averages) is about 2°-3°.
- As in any system, other sources of error include hardware failure, misalignments, or misuse.
- **Direction finding -- although unconventional and probably not useful for microwave radars -- is ideally suited to HF coastal current mapping, offering far more advangages than phased arrays that employ beam forming.**

FAQ -- How Does SeaSonde Measure Direction with Its Compact Antennas?

Q. Your Codar systems have three colocated, tiny compact antennas with very broad beams at least 90° wide. You're telling me that from their three signals, you can get current vector bearings to better than 5°? Do you make some monstrous assumption (like fitting a restrictive model to the data), or do I have to wade through pages of heavy math in esoteric radar physics to understand how this miracle happens?

A. None of these; you only have to be familiar with the 'tangent' function in trigonometry. Our systems use direction finding (DF), perhaps the oldest known method of measuring the bearing of radio signals that goes back to 1900, far before radar was ever heard of. DF was used long before beam forming, that employs antennas many wavelengths in extent (an array at HF a hundred meters or more long). There are three small antennas in the receive unit: two crossed loops and a monopole. Each receives vertically polarized radio signals, which is the only polarization component that can propagate above the highly conducting sea. The two loops have cosine patterns ('figure-8') at right angles to each other; the monopole has an omni-directional pattern. Sketches of these patterns are shown below when looking offshore from a straight coastline. When antenna amplitudes and phases are calibrated properly, the loops respond to a signal of complex amplitude **S** according to the equations shown below. Therefore, a signal coming from direction ϑ at a frequency f will have the responses shown.

Loop #1 Voltage Response to Signal **S** From Direction ϑ

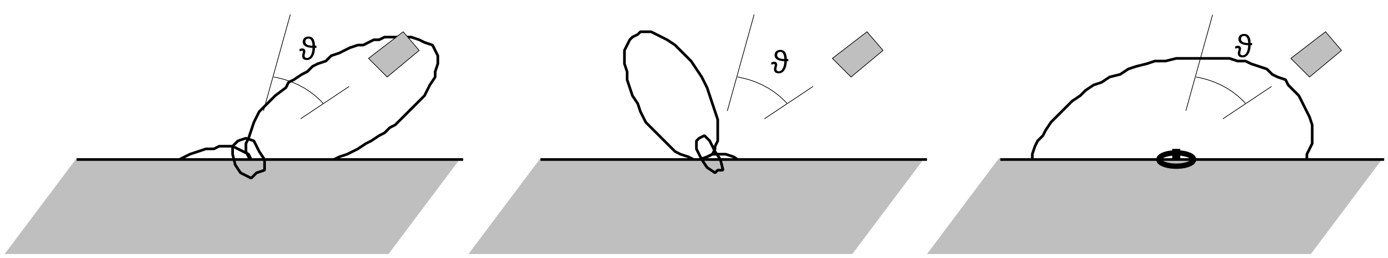
$$\mathbf{v_1} = \mathbf{S}\ \cos(\vartheta - 45°)$$

Loop #2 Voltage Response to Signal **S** From Direction ϑ

$$\mathbf{v_2} = \mathbf{S}\ \cos(\vartheta + 45°)$$

Monopole Voltage Response to Signal **S** From Direction ϑ

$$\mathbf{v_3} = \mathbf{S}$$



For Perfect Antenna Patterns

$$\vartheta(f) = \mathrm{ATAN2}\,[\mathbf{v_1}(f)\,/\,\mathbf{v_3}(f),\ \mathbf{v_2}(f)\,/\,\mathbf{v_3}(f)\,] - 45°$$

The bearing angle ϑ is obtained by dividing the voltages of the loops by the monopole signal -- thereby cancelling the unknown and irrelevant signal amplitude **S** -- and the 'Arctangent2' function is taken (in software). This works for signals coming from 360° in bearing; the monopole serves as the 'quadrant' reference in order to resolve the 180° ambiguity that loops alone have.

Q. Wait a minute! This is all fine if I have only one signal coming into my antennas. What if I have two signals? What happens then?

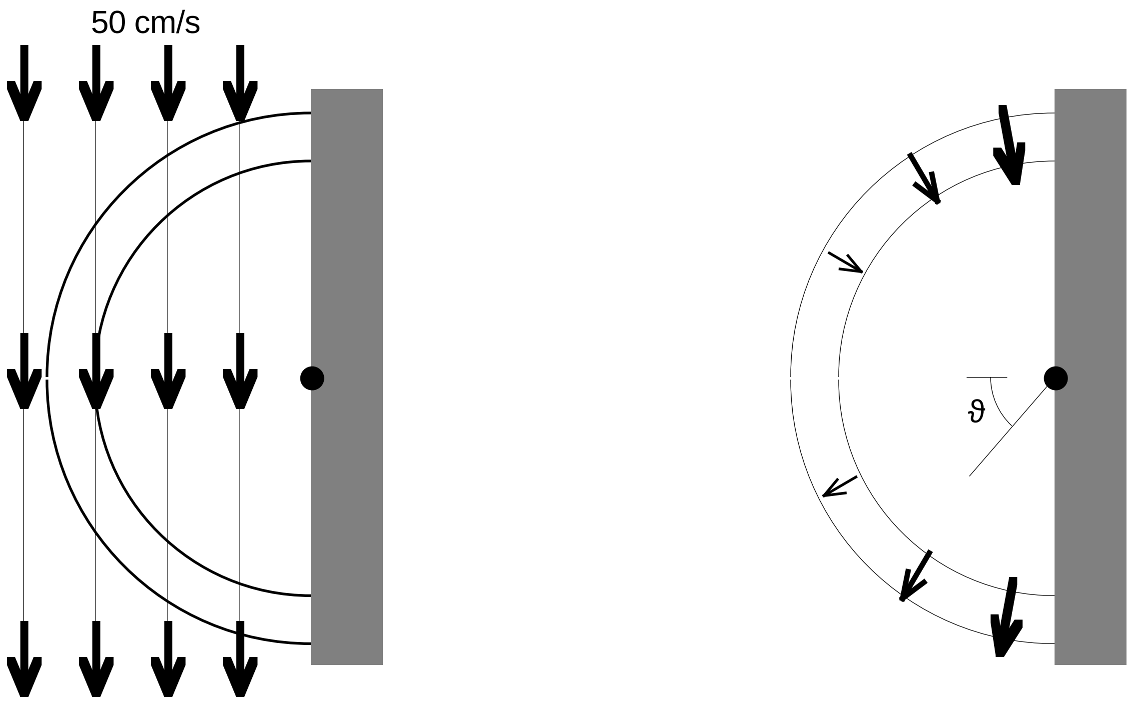
A. No problem with two signals. You can uniquely resolve two signals. A proof is given in the Appendix for those interested in rigor. Consider the following explanation for now. There are three received complex voltages, $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$. These constitute six measured real observables. With two signals, there are two unknown complex signal amplitudes $\mathbf{S_1}$ and $\mathbf{S_2}$; as well as two real unknown bearing angles, ϑ_1 and ϑ_2. Hence there are six real unknowns and six measured quantities. Therefore, if you do not want to read the derivation in the Appendix, perhaps you might accept that one can uniquely solve for the two desired unknown bearing angles, ϑ_1 and ϑ_2. If you want a bit of mental exercise, follow through the proof in the Appendix.

Q. OK, fine, but it appears to me that we really have scattered signals coming from all over the ocean, not just from one or two directions. This is where I still have a problem. The system is receiving continuous signals from many bearings, and so far you have only described how your simple antenna can resolve one angle, or at most, two. So you have a ways to go.

A. Follow through this analogy. In the simplest and oldest of direction-finding systems, I would mechanically turn the loop to the null position. I would thereby determine the bearing of the radio signal. When there are many signals in the air, here's how I go about finding the direction of each. If they come from different stations, they are at different frequencies. I merely tune my receiver to each frequency, one at a time, and rotate the loop null axis until it points in the direction of each signal source. Thereby I have found the bearings of many signals from different directions.

Q. Wait a minute! We're not talking about signals at different frequencies here. We're talking about continuous sea scatter coming from many bearings simultaneously, perhaps from a sector as much as 180° looking out from the coast. What does this have to do with radio stations with different frequencies? Be honest: are you fitting some sort of hydrodynamic model to the real situation, or are we expected to believe in miracles here?

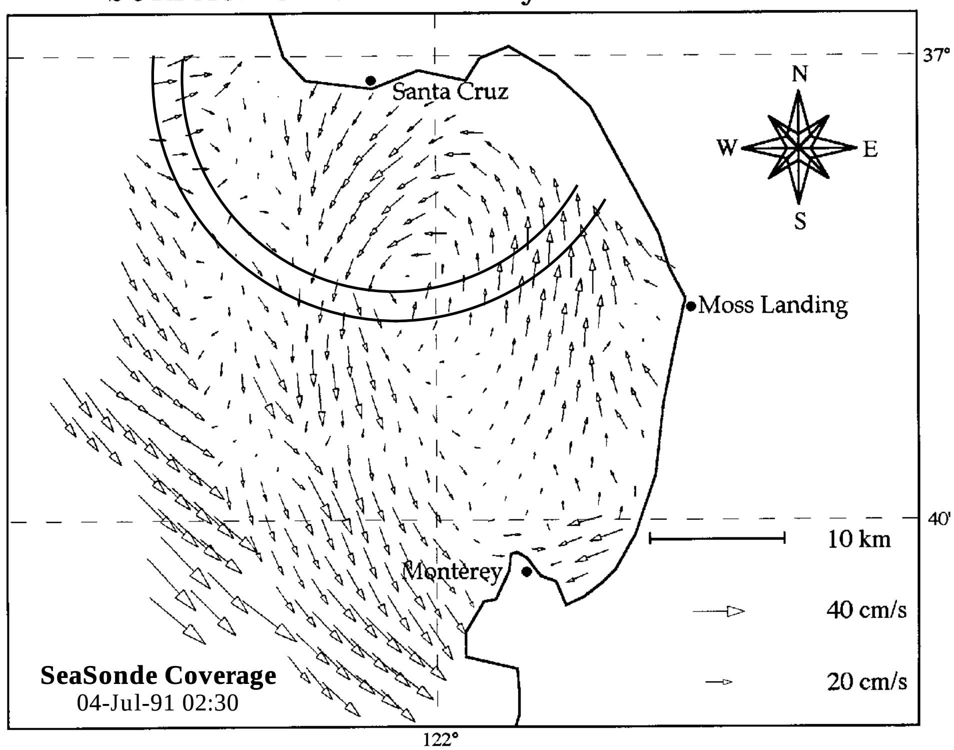
A. Not at all. We really are dealing with signals at *different* frequencies, just like our signals from multiple radio stations. Take a simple example first: we have a straight coastline and a uniform flow parallel to the coast as shown below. Remember that *any* radar -- through the Doppler relation -- sees only the component of velocity pointing toward (or away from) the radar, i.e., the radial component. So we spectrally analyze the received signal (do an FFT on the signal voltages). Consider the SeaSonde with its 12 MHz operating frequency. Assume our uniform current has an alongshore velocity of 50 cm / s (about one knot, a rather modest current). In our normal processing, the time series used for spectral analysis is 256 seconds long. This gives a spectral resolution of 4 millihertz, corresponding to Doppler speed resolution of slightly less than 5 cm / s. What all this actually means is that the output of the FFT is a bunch of signals at different frequencies. *This is identical to having a receiver that could tune to separate signal frequencies digitally, in 4 millihertz frequency steps (or 5 cm/s radial velocity steps).* Let's see how many different signal radio frequencies we are 'tuning to' by our digital signal spectral analysis. The radial velocity ranges between +50 cm / s to the North to -50 cm / s to the South. The exact radial velocity at an angle ϑ is $v_r = 50 \sin\vartheta$ (cm / s), and the received frequency vs bearing from the Doppler relation is $f_D = 0.04 \sin\vartheta$ (Hz). We have a total span of 100 cm / s divided by 5 cm / s per bin, *giving 20 different signal frequencies, each of whose directions can now be found uniquely*. And this is only for *one* of the two Bragg echo peaks (representing scatter from waves, for example, approaching the radar). When one includes the information from the other peak (which is always present), this number of independent signals can double. Therefore, we can have 40 different bearings available by applying the Arctangent function to each of these signals. For the alongshore flow shown, this would lead to between 20 and 40 different directions. Over 180°, these on average are spaced at 4.5° to 9° apart. So this is quite a good picture of the flow over a given range cell, is it not?

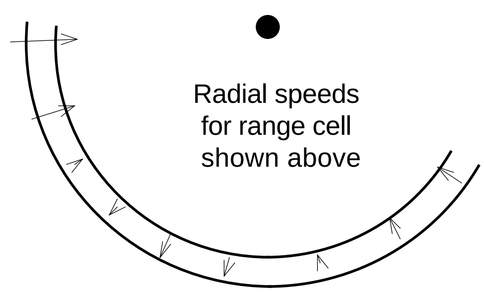


Q. Fine, fine! But you've picked a rather idealized, unrealistic flow for your example; if all currents were this simple, we wouldn't need HF radars to measure them. What about a more complex circulation pattern?

A. Shown below is a rather complex, interesting pattern that recurs frequently in Monterey Bay, as seen by the SeaSonde network. Whether or not you want to accept at this stage that the SeaSonde can measure it accurately, admit that it is complex. I'm drawing a range cell through it from our radar at Santa Cruz, in a region where the pattern appears to undergo the most variation. Underneath the total-vector current map is a sketch of the radial speeds that are sampled by the radar for this range cell.

SURFACE CIRCULATION of MONTEREY BAY





The radial currents vary continuously from strong approaching toward the West, to zero SouthWest, to mild receding toward SouthSouthWest, to zero South, to mild approaching to the SouthEast. Consider the number of angle directions with the same radial speeds: there is generally only one, but at most two angles over this fairly complex current structure. Recall that the crossed-loop / monopole receive antennas and subsequent algorithm can resolve signals at the same frequency (speed) from either one or two different directions in the same velocity / Doppler bin. There are two bins with the same velocity, corresponding to the two Bragg peaks (redundant information); allowing for up to four angles with the same velocity. Go ahead and try placing circular range cells at other locations, and see if you can find any situations where there might be more than two. I can't. *Critics have claimed you can't measure complex spatial patterns -- like eddies and gyres -- with simple crossed-loop direction finding. Haven't I just proved you can? Do you detect any hidden magic in these simple arguments?*

Q. You're saying that most realistic -- and even quite complex -- current patterns can be handled with this technique, because they have at most two different directions that produce the same radial speed (or received signal frequency)?

A That's what I'm saying alright. There may possibly be a few cases with three. What happens then is this. First of all, the algorithm we use now is called 'MUSIC' (MUltiple SIgnal Classification), which is based on an eigen-analysis of the covariance matrix between the three antenna elements. Prior to this, we successfully used a 'maximum-likelihood' method, which is a variation on 'least squares'. These algorithms take the noise level into account, and based on statistical confidence testing, they decide whether the bearing at a given radial speed best satisfies a one or two angle situation. Thereby they select the appropriate one. The amplitudes associated with the signals from one, two, (or perhaps three?) bearings will not be equal, so in the rare event there happens to be three, it will 'home in' on the directions of the two strongest signals. A gap may appear at the position of the third signal. But the amplitude of the signal at that frequency from the other Bragg peak will typically be stronger (the receding waves doing the scattering are most likely stronger if the approaching waves were weak at that point), so the gap (or third angle) will always be filled from the 'redundant' information available from the other peak. We have done simulations, where the inputs are known, having a quadruple ambiguity (four angles with the same speed), and obained a pretty good recovery of the input current. [*Oceanography*, vol. 10, no. 2, pp. 72-75 1997]

Q. What about phased arrays? Won't they do a lot better job?

A. They have both limited resolution and field of view. At ~12 MHz (one of the four SeaSonde frequency bands), a phased-array receive antenna 100 m in length (quite a bit of coastal real estate, you must admit, not even counting the transmit antenna) will have an average beamwidth varying between 14° straight out, to 20° at the edges of its ±45° maximum field of view. This gives about six independent beams in this 90° scan quadrant. That's not a lot of bearing information. Of course, some have foolishly argued that you can scan the beam to *any* angle grid (thousands of angle bins) by just setting the phases (to thousands of settings), but the inherent information content remains dictated by its resolution: *six pieces of angle information*. To resolve a quadruple-valued radial current pattern, you would need eight beams in this sector, by the familiar Nyquist criterion. Longer arrays will of course do better, but consume commensurately greater dollars and land.

Q. Wait a minute! I've got a case I'll bet you can't handle with direction finding. Suppose the currents are very weak, so your 50 cm/s becomes only 5 cm/s maximum. Since the 'bins' -- containing the different signal Doppler frequencies resulting from the scatter -- are 5 cm/s wide, there are now *only two* between +5 cm/s and -5 cm/s (centered on +2.5 cm/s and -2.5 cm/s). So there may be as few as only two bearing directions over 180° of angle space. Not very good angle coverage or resolution! Have I found your Achilles heel?

A. Not really. We know the speeds for those two bins very precisely. And we know that's all there are: just two bins, because the 'width' of the Bragg peaks (i.e., the number of bins containing current signals) is very well determined. Suppose the two angle solutions fall out at -57° and +29°, over our 180° sector. We know that the currents between these points (and everywhere else over this sector) *are no greater* than + 2.5 cm/s and -2.5 cm/s. So we can interpolate in bearing-angle space to get a smooth plot of radial speed vs angle. If this wide-angle interpolation bothers you, remember, *the most you can be off at any bearing point is ±2.5 cm/s,* which is merely the Doppler resolution of the system. Simple physics and reasoning!

Q. OK, I accept that you can get direction from your simple tiny SeaSonde antenna system. But how accurately?

A. Estimation of accuracy in DF systems is a well known process. Accuracy depends on the noise level. We estimate the signal covariance matrix throughout the processing, which is a measure of the noise power and its impact on the three antenna signals. As the simplest rule of thumb, however, the rms angle error in radians is inversely proportional to the square root of the signal-to-noise ratio, S/N times the number of independent samples, n, i.e., $(n_x S/N)^{-1/2}$. For example, suppose S/N is 100 (i.e., 20 dB), and n = 14 samples are averaged (this is exactly the case with the SeaSonde at 256 seconds per sample over one hour). This gives an rms bearing error of 0.0267 radians or 1.53°. In 1980 measurements at Duck, NC, we established a statistical angle error between 2° and 3° from the fluctuation in the data, for slightly less averaging. These two numbers are therefore in reasonable agreement.

APPENDIX: Determination of Two Directions from Crossed-Loop/Monopole Signals

Assume two signals (at the same frequency) arrive at the co-located crossed loops and monopole, from unknown directions ϑ_a and ϑ_b, with complex unknown amplitudes $\mathbf{S}_a$ and $\mathbf{S}_b$. Assume here for convenience that the angles are defined with respect to the Loop #1 axis. Then the complex received voltages are:

$$\mathbf{v}_1 = \mathbf{S}_a \cos\vartheta_a + \mathbf{S}_b \cos\vartheta_b ;$$
$$\mathbf{v}_2 = \mathbf{S}_a \sin\vartheta_a + \mathbf{S}_b \sin\vartheta_b ;$$
$$\mathbf{v}_3 = \mathbf{S}_a + \mathbf{S}_b .$$

Solve the first two of these equations for $\mathbf{S}_a$ and $\mathbf{S}_b$ in terms of the measured loop voltages and the angles, to get;

$$\mathbf{S}_a = \frac{\mathbf{v}_1 \sin\vartheta_b - \mathbf{v}_2 \cos\vartheta_b}{\sin(\vartheta_b - \vartheta_a)} \quad \text{and} \quad \mathbf{S}_b = \frac{\mathbf{v}_1 \sin\vartheta_a - \mathbf{v}_2 \cos\vartheta_a}{\sin(\vartheta_a - \vartheta_b)} .$$

Substitute these into the third equation for the monopole voltage to get:

$$\mathbf{v}_3 = F_1 \mathbf{v}_1 + F_2 \mathbf{v}_2 , \text{ where}$$
$$F_1 \equiv \frac{\sin\vartheta_a - \sin\vartheta_b}{\sin(\vartheta_a - \vartheta_b)} = \frac{\cos\left(\frac{\vartheta_a + \vartheta_b}{2}\right)}{\cos\left(\frac{\vartheta_a - \vartheta_b}{2}\right)} ;$$
$$F_2 \equiv \frac{\cos\vartheta_b - \cos\vartheta_a}{\sin(\vartheta_a - \vartheta_b)} = \frac{\sin\left(\frac{\vartheta_a + \vartheta_b}{2}\right)}{\cos\left(\frac{\vartheta_a - \vartheta_b}{2}\right)} .$$

Note that F_1 and F_2 are pure real, but the equation above is complex. Therefore, it becomes two equations in the two unknowns: F_1 and F_2.

$$\mathrm{Re}[\mathbf{v}_3] = F_1 \mathrm{Re}[\mathbf{v}_1] + F_2 \mathrm{Re}[\mathbf{v}_2] ,$$
and
$$\mathrm{Im}[\mathbf{v}_3] = F_1 \mathrm{Im}[\mathbf{v}_1] + F_2 \mathrm{Im}[\mathbf{v}_2] .$$

This is then solved for the two unknowns, F_1 and F_2, from which we obtain the following equations for the two unknown bearing angles of the signals:

$$\vartheta_a + \vartheta_b = 2\mathrm{ATAN2}(F_2, F_1) ,$$
and
$$\vartheta_a - \vartheta_b = 2\mathrm{ACOS}\left(\frac{\cos\frac{\vartheta_a + \vartheta_b}{2}}{F_1}\right) .$$

Thus we have solved uniquely for the two angles at the same frequency. (If you still doubt, try it; it's a good 'pocket calculator' exercise. Dream up signals from two different bearings, ϑ_a and ϑ_b, having any arbitrary complex amplitudes $\mathbf{S}_a$ and $\mathbf{S}_b$. Put these into the first three equations in the Appendix to get the complex voltages received by the antennas. Then follow the subsequent steps to retrieve the angles. It works!)

Simple Method to Distinguish Dual-Angle from Single-Angle Case: If there is only a single signal present from one angle, we have as our voltages and our solution:

$$\mathbf{v}_1 = \mathbf{S} \cos\vartheta \; ;$$
$$\mathbf{v}_2 = \mathbf{S} \sin\vartheta \; ;$$
$$\mathbf{v}_3 = \mathbf{S} \; .$$

$$\vartheta = \mathrm{ATAN2}\left(\frac{\mathbf{v}_2}{\mathbf{v}_3}, \frac{\mathbf{v}_1}{\mathbf{v}_3}\right) .$$

Note, however, that for the single-angle case, we clearly have the following identity:

$$|\mathbf{v}_3|^2 = |\mathbf{v}_1|^2 + |\mathbf{v}_2|^2 \; .$$

By examining the first three equations of the Appendix for dual-angle voltages -- and trying a couple examples -- you can convince yourself that the above identity is not true when two signals are present. Therefore, this identity is the simplest test (but not necessarily the most robust statistically) of whether one or two signals from different angles are present.